VERBAL SUBGROUPS OF GROUPS

Research program

Let $w = w(x_1, \ldots, x_k)$ be a group-word, that is, a nontrivial element of the free group F with free generators $x_1, x_2, \ldots$. The verbal subgroup $w(G)$ of a group G determined by w is the subgroup generated by the set G_w consisting of all values $w(g_1, \ldots, g_k)$, where $g_1, \ldots, g_k$ are elements of G. It is interesting to study how properties of the set of w-values influence the structure of $w(G)$. For example, if G_w is finite, does it follows that $w(G)$ is also finite?

In this context, a word w is said to be concise if whenever G_w is finite for a group G, it always follows that $w(G)$ is finite. More generally, a word w is said to be concise in a class of groups $\mathcal{X}$ if whenever G_w is finite for a group $G \in \mathcal{X}$, it always follows that $w(G)$ is finite. P. Hall asked whether every word is concise, but later Ivanov proved that this problem has a negative solution in its general form [7] (see also [8, p. 439]). On the other hand, many relevant words are known to be concise. In particular, it was shown in [9] that the multilinear commutator words are concise. One of the results obtained in [2] says that for any multilinear commutator word $w = w(x_1, \ldots, x_k)$ and any positive integer n the word $[w, {}_n y]$ is concise in residually finite groups.

A word w is boundedly concise in a class of groups $\mathcal{X}$ if for every integer m there exists a number $\nu = \nu(\mathcal{X}, w, m)$ such that whenever $|G_w| \leq m$ for a group $G \in \mathcal{X}$ it always follows that $|w(G)| \leq \nu$. Fernandez-Alcober and Morigi showed that every word which is concise in the class of all groups is actually boundedly concise [5]. It was conjectured in [6] that every word which is concise in the class of residually finite groups is boundedly concise. In [3, 4] various Engel type words have been proved to be concise or boundedly concise in the class of residually finite groups, but the question of bounded conciseness of words in profinite groups is still open.

In the class of profinite groups a strengthened version of Hall's conciseness conjecture has been suggested, namely if every word is strongly concise in the class of profinite groups. A word w is said strongly concise in a class $\mathcal{X}$ of topological groups if whenever G is a group in $\mathcal{X}$ and G_w has less tha $2^{\aleph_0}$ elements it always follows that $w(G)$ is finite. In [1] multilinear cummutator words are proved to be strongly concise in the class of profinite groups, as well as the words: $x^2, x^3, x^6, [x, y, y]$, $[x^2, z_1, \cdots, z_r]$, $[x^3, z_1, \cdots, z_r]$, $[x, y, y, z_1, \cdots, z_r]$, where $x, y, z_1, z_2, \ldots$ are independent variables.

The candidate is expected to work in the above research subjects. In particular the following problems cam be dealt with:

1) Investigate conciseness of words in the class of residually finite groups, trying to determine if all words that are known to be concise in the class of residually finite groups are also boundedly concise.

2)) Investigate strong conciseness of words in profinite groups. This is a new and challenging problem, and even partial results for the word x^4 or for some particular Engel words in pro-p-groups would we significant in this contest.

In the first 3 months the candidate will study possible missing prerequisites to face the above research problem. In the remaining 9 months the candidate is expected to actively participate to the research project.

References

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[4] E. Detomi, M. Morigi, P. Shumyatsky, Words of Engel type are concise in residually finite groups. Part II, *Groups Geom. Dyn.*, to appear, arXiv:1905.07784.

[5] G. A. Fernández-Alcober, M. Morigi, Outer commutator words are uniformly concise. *J. London Math. Soc.* **82** (2010), 581–595.

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